NACS 643 Homework #4
Spring 2009

Notes on solving ordinary differential equations in Matlab. There are several extremely powerful ways to solve ordinary differential equations in Matlab, which are fine to use if you know them, but these steps (the Euler method) will get you by otherwise.

A) We'll start with an equation in the form $\tau \frac{du}{dt} = -u + C + f_{\text{ext}}(t)$. For instance, $\tau$ is a time constant, $u$ is the variable of interest (e.g. voltage), $C$ is a constant (e.g. a reversal potential), and $f_{\text{ext}}(t)$ is an external driving force (e.g. proportional to an applied current) that you know.

B) Choose a small time step $\Delta t$ (much smaller than $\tau$) so we can make time discrete: $t_k = k\Delta t$, and we'll approximate $\dot{u} = \frac{du}{dt}$ by $\frac{u(t) - u(t-\Delta t)}{\Delta t}$. That is $\dot{u}(t_{k+1}) = \frac{u(t_{k+1}) - u(t_k)}{\Delta t}$, abbreviated as $\dot{u}_{k+1} = \frac{u_{k+1} - u_k}{\Delta t}$.

C) The goal is to generate $u$ at each time step based on its value (and the value of the external input) at previous time steps, and do it in a way that is stable against numerical artifacts. That is, we calculate $u_{k+1}$ from $u_k$, $f_{\text{ext}}(t_k)$, $\Delta t$, and the other constants.

D) Rearranging the derivative equation, we get $u_{k+1} = u_k + \dot{u}_{k+1}\Delta t$, which is almost what we need.

E) Going back to the original equation in (A) see that $\dot{u}_{k+1} = \left(-u_k + C + f_{\text{ext}}(t_k)\right)/\tau$, is an obvious choice.

F) To summarize, using $u_{k+1} = u_k + \Delta t\left(-u_k + C + f_{\text{ext}}(t_k)\right)/\tau$ will generate solutions of the equation in (A), one time step at a time.

G) The method described here is very general: if the equation in (A) is changed, then the same algorithm applies: use step (D) "as is", but modify step (E) for the new equation.

1) Build a model integrate-and-fire neuron using the equation $\tau_m \frac{dV}{dt} = -V + E_L + R_m I_e$. Use $R_{\text{m}} = 10\ \text{M}\Omega$, and $\tau_{\text{m}} = 10\ \text{ms}$. Initially set $V$ to $V_{\text{rest}} = E_L = -70\ \text{mV}$. When the membrane potential reaches $V_{\text{th}} = -54\ \text{mV}$, make the neuron fire a spike and reset the potential to $V_{\text{reset}} = -80\ \text{mV}$.

   a) Show sample voltage traces (with spikes) for a 300-ms-long current pulse (choose a reasonable current $I_e$) centered in a 500-ms-long simulation.

   b) Determine the firing rate of the model for various magnitudes of constant $I_e$ and and compare the results with the formal interspike-interval firing rate: $r_{\text{isi}} = \frac{1}{t_{\text{isi}}} = \left[\tau_m \ln\left(\frac{R_m I_e + E_L - V_{\text{reset}}}{R_m I_e + E_L - V_{\text{th}}}\right)\right]^{-1}$.

2) Add an excitatory synaptic conductance to the integrate-and-fire neuron of the first problem by adding the extra synaptic conductance term: $\tau_m \frac{dV}{dt} = -V + E_L - r_m \overline{g}_s P_s (V - E_s) + R_m I_e$. Using $E_s = 0$, set the external current to zero, $I_e = 0$, in this example, and assume that the probability of release on receipt of

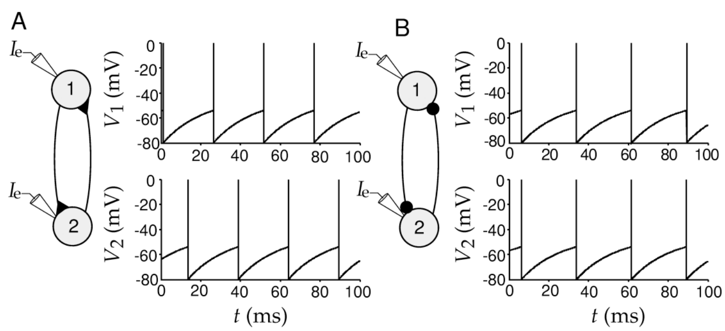
a presynaptic spike is 1. Use $E_L = -70$ mV and $r_m \bar{g}_s = 0.5$ and describe $P_s$ using an alpha function: $P_s \propto t \exp(-t/\tau_s)$. More specifically, we'll use Dayan & Abbot's version of the equation, $P_s = P_{\max} t/\tau_s \exp(1 - t/\tau_s)$ with $\tau_s = 10$ ms and $P_{\max} = 0.5$.

a) Trigger synaptic events at times 50, 150, 190, 300, 320, 400 and 410 ms. Plot $V(t)$ in one graph and the synaptic current (or, more simply, $r_m \bar{g}_s P_s (V - E_s)$) in another. Explain what you see.

b) Optional: To *correctly* incorporate multiple presynaptic spikes, $P_s$ should be described by a pair of differential equations,

$$\tau_s \frac{dP_s}{dt} = -P_s + eP_{\max} z(1 - P_s) \text{ and } \tau_s \frac{dz}{dt} = -z$$

where $e = \exp(1)$, with the additional rule that $z$ is set to 1 whenever a presynaptic spike arrives. Repeat part (a) using this description of the synaptic conductance instead of the naïve alpha function. Explain the differences.




3) Construct a model of two coupled integrate-and-fire neurons similar to that in figure 5.20 of Dayan & Abbot (and shown above). Both model neurons obey $\tau_m \frac{dV}{dt} = -V + E_L - r_m \bar{g}_s P_s (V - E_s) + R_m I_e$ with $E_L = -70$ mV, $V_{\text{th}} = -54$ mV, $V_{\text{reset}} = -80$ mV, $\tau_{\text{m}} = 20$ ms $r_m \bar{g}_s = 0.15$, and $R_m I_e = 18$ mV. Both synapses should be described as in problem 2 with $P_{\max} = 0.5$ and $\tau_s = 10$ ms. Consider cases where both synapses are excitatory, with $E_s = 0$, and both are inhibitory, with $E_s = -80$.

a) Show how the pattern of firing for the two neurons depends on the type (excitatory or inhibitory) of the reciprocal synaptic connections. For these simulations, set the initial membrane voltages of the two neurons to slightly different values, randomly, and run the simulation until an equilibrium situation has been reached, which may take a few seconds of simulated run time. Start from a few different random initial conditions to study whether the results are consistent.

b) Optional: Investigate what happens if you change the strengths and time constants of the synapses.

c) Optional: Correctly implement the time course of the synaptic conductances as in problem 2b.